The response of a layered electron gas to a transverse electromagnetic field

1996 J. Phys.: Condens. Matter 8 7745

(http://iopscience.iop.org/0953-8984/8/41/019)

J. Phys.: Condens. Matter **8** (1996) 7745–7754. Printed in the UK

The response of a layered electron gas to a transverse electromagnetic field

Ami C Sharma†‡ and S Peter Apell‡
† Physics Department, Faculty of Science, The MS University of Baroda, Vadodara-390 002, India
‡ Department of Applied Physics, Chalmers University of Technology and Göteborg University, S-41296 Göteborg, Sweden

Received 10 January 1996, in final form 12 August 1996

Abstract. The propagation of transverse electromagnetic waves in a layered electron gas is studied from the point of view of the electromagnetic penetration depth. It is found that transverse in-plane plasmons can be observed in systems consisting of *two* layers of charge carriers per unit cell for special values of wave vector and frequency, determined by the intrinsic parameters of the set-up like the Fermi velocity and the length scales of the unit cell. The internal degrees of freedom in the two layers per unit cell as compared to one layer per unit cell make possible this acoustic-like transverse plasmon. Above the cut-off frequency transverse electromagnetic waves are attenuated as they propagate in the layered electron gas. The layered gas furthermore behaves like a three-dimensional free-electron gas when the layers are close, having the same penetration depth dependence on frequency and the electron relaxation time. In the opposite limit of remote layers the penetration depth is found to be almost independent of these parameters.

1. Introduction

The study of various aspects of layered electron gases (LEG) is of great importance to the understanding of the properties of a class of novel materials such as modulation-doped compositional and strained semiconductor superlattices, doping semiconductor superlattices and cuprate superconductors. The semiconductor superlattices and the single-crystal cuprate superconductors can be modelled as one-dimensional arrays of two-dimensional (2D) electron/hole sheets embedded into dielectric (host) media. There have been extensive theoretical investigations on semiconductor superlattices and cuprate superconductors using the LEG model [1–21]. The response of a LEG to an electromagnetic field has also been a subject of interest from the viewpoints of propagation of electromagnetic waves and the coupling of fields to charge fluctuations in the LEG. There exists a considerable literature on plasmons, plasmon–phonon coupled modes and the light scattering in semiconductor superlattices and superconductors [1–3, 10–22]. These investigations have been motivated by experiments on epitaxially grown semiconductor superlattices [23–26] and the superconducting superlattices [27]. However, in most existing investigations, the response of a LEG to nonretarded longitudinal electromagnetic (LEM) fields has been considered [1]. There have been few theoretical studies which consider the response of a LEG to both longitudinal and transverse electromagnetic (TEM) fields in the retarded limit [2, 3, 6].

From the existing literature [1–27], one reaches the understanding that the free propagation of TEM waves can take place in a 3D electron gas and LEG in the retarded limit, i.e. where $\omega \gg kc$, while LEM waves can freely propagate in an electron gas mainly for $\omega \ll kc$. ω, k and c are the frequency, wave vector and velocity of light respectively. In the nonretarded limit, i.e. where $\omega \ll kc$, TEM waves are attenuated as they travel through the electron gas. The distance over which attenuation takes place is called the penetration depth or skin depth, depending upon the ratio γ/ω. γ is the inverse of the relaxation time of a charge carrier.

The aim of this paper is to show that TEM waves can also propagate freely in a LEG in the limit where $\omega \ll kc$, if we construct a system consisting of two conducting layers of charge carriers in a unit cell. These transverse plasmons (self-sustained freely propagating TEM waves) can be produced in this class of materials for $\omega \leqslant \omega_c$ and $k \leqslant k_c$, where ω_c and k_c are upper cut-off values of ω and k, respectively. The frequency of the transverse plasma mode is found to be in the microwave regime or higher, using relevant parameters for a typical superlattice. Such a system, consisting of two layers of charge carriers in a unit cell, might be a modulation-doped GaAs/$Al_xGa_{1-x}As$ superlattice with two unequal successive layers of $Al_xGa_{1-x}As$ which are separated by a GaAs layer, a type-II compositional or strained semiconductor superlattice, a doping superlattice, or a cuprate superconductor like $YBa_2Cu_3O_{7-\delta}$ or $Bi_2Sr_2CaCu_2O_8$. We further demonstrate that for $k > k_c$ and $\omega_c < \omega \ll kc$, TEM waves are attenuated inside the LEG.

In what follows we calculate the penetration depth for TEM waves in a system with one electron layer per unit cell to introduce our notation and the connection to the more well-known case of penetration in a 3D homogeneous electron gas. The main emphasis in the paper is on a field traversing the material perpendicularly to the stacking direction. After that we do a similar calculation for a system consisting of two conducting layers of charge carriers within a unit cell. We then find the possibility of having transverse plasmons propagating as well as a frequency region where the field does not penetrate completely. In the last section we introduce dispersion into our dielectric description to find the frequencies of the new transverse plasmon modes.

2. General relations

We consider a one-dimensional (1D) array of two-dimensional (2D) conducting sheets embedded along the z-axis into a dielectric medium of static dielectric constant ϵ_b. In the absence of an external current source, the condition for propagation of a TEM wave in the x–y plane is given by the solution to the equation

$$1 + 4\pi \mathrm{i}\omega R(q, k_z, \omega)\sigma_t^0(q, \omega) = 0 \tag{1}$$

from Shi and Griffin [3]. q is the component of $\boldsymbol{k}$ in the x–y plane. k_z is a component along the z-axis. $\sigma_t^0(q, \omega)$ is the transverse component of the general conductivity tensor of a homogeneous 2D electron gas.

R is a structure factor of the LEG which has the following forms:

$$R(q, k_z, \omega) = \frac{1}{d(\omega^2 - c^2q^2 - c^2k_z^2)} \tag{2}$$

when $\omega > qc$ and

$$R(q, k_z, \omega) = \frac{-1}{2pc^2}\left[\frac{\sinh(pd)}{\cosh(pd) - \cos(k_zd)}\right] \tag{3}$$

when $\omega < qc$. $p^2 = q^2 - (\omega/c)^2$, and d is the length of a unit cell along the z-axis of the LEG.

In our calculations of the penetration depth we will use the following long-wavelength-limit result ($qv_F \ll \omega \ll qc$) for the conductivity [3]:

$$\sigma_t^0(q,\omega) = \frac{\mathrm{i}d\omega_p^2}{4\pi\omega} + \mathrm{O}\left[\frac{q^2}{\omega^3}\right]. \tag{4}$$

$\omega_p^2 = 4\pi n_s e^2/m^*\epsilon_b d$ is the average plasma frequency including the screening from an embedding background with dielectric function ϵ_b. n_s is the charge density per unit area and m^* is the effective mass of the charge carrier.

In analogy with the definition of a longitudinal dielectric function in [3, 28] we introduce a transverse dielectric function for the LEG:

$$\epsilon_t(q,k_z,\omega) \equiv 1 + \frac{4\pi\mathrm{i}}{\omega} R(q,k_z,\omega)(\omega^2 - c^2q^2)\sigma_t^0(q,\omega). \tag{5}$$

In terms of ϵ_t we can rewrite equation (1) in a form analogous to the bulk polariton dispersion relation for a 3D solid:

$$q^2 = \frac{\omega^2}{c^2}\epsilon_t(q,k_z,\omega). \tag{6}$$

Notice that equation (6) is equivalent to equation (1); however, this way of writing things makes it easier to insert the appropriate dielectric function in our subsequent analysis.

In what follows we will use the following long-wavelength form of ϵ_t for one layer per unit cell:

$$\epsilon_t(q,k_z,\omega) = 1 - \frac{\omega_p^2 d}{\omega(\omega+\mathrm{i}\gamma)} R(q,k_z,\omega)(\omega^2 - c^2q^2) \tag{7}$$

by inserting equation (4) into equation (5) and making the substitution $\omega^2 \to \omega(\omega+\mathrm{i}\gamma)$ when we have a scattering mechanism present, through a relaxation time $1/\gamma$. In the nonretarded limit ($\omega < qc$), $R(q,k_z,\omega)(\omega^2 - c^2q^2)$ reduces to $\frac{1}{2}pS(p,k_z)$ where $S(p,k_z)$ is defined as

$$S(p,k_z) = \frac{\sinh(pd)}{\cosh(pd) - \cos(k_z d)}. \tag{8}$$

If $\omega \ll qc$ we can set $p \equiv \sqrt{(q^2 - (\omega/c)^2)} \simeq q$.

The corresponding transverse dielectric function for the system having two layers of charge carriers in a unit cell can, for $\omega < qc$, be written as [17]

$$\epsilon_t(q,k_z,\omega) = 1 - \frac{(\omega_{p1}^2 + \omega_{p2}^2)qdS(q,k_z)}{2\omega(\omega+\mathrm{i}\gamma)} + \frac{\omega_{p1}^2\omega_{p2}^2(qd)^2S(q,k_z)f(q)}{4\omega^2(\omega+\mathrm{i}\gamma)^2} \tag{9}$$

where

$$f(q) \equiv \frac{\cosh(qd) - \cosh(q(2d_1 - d))}{\sinh(qd)}. \tag{10}$$

ω_{p1} and ω_{p2} are obtained from ω_p on replacing n_s by n_{s1} and n_{s2} respectively. n_{s1} and n_{s2} are carrier densities per unit area in the x–y planes of layer one and two respectively, in a unit cell. d_1 is the distance between these two layers. The third term on the right-hand side of equation (9) originates from the *interlayer* interaction in a unit cell, while the second term on the right-hand side of equations (7) and (9) corresponds to *intralayer* interactions. On top of this, $S(q,k_z)$ and $f(q)$ contain the interactions between all of the cells in the system.

3. The penetration depth along the planes

In this section we calculate the penetration depth of a TEM field which travels along the planes in the LEG with the field vector in the planes. We then assume k_z to be real and the LEG to be transparent along the z-axis. However, if we solve equation (6) above, considering k_z to be complex and q to be real, we then get penetration of a TEM field along the z-axis instead of in the x–y plane. In this case the LEG is transparent within the x–y plane. The penetration depth along the z-axis is in this case found to be roughly $1/q$ for all values of γ/ω since there cannot be any energy transfer to electrons along the z-axis. We therefore consider only the more interesting case of a TEM field in the plane of the layers in what follows. However, first we will introduce some notation from the well-known 3D situation.

If we take equation (7) and insert $R(q, k_z, \omega)$ from equation (2) ($\omega > qc$) it is easy to demonstrate that the following is a solution to equation (6), for small γ:

$$\omega^2 = \omega_p^2 + c^2(q^2 + k_z^2). \tag{11}$$

This is simply the bulk polariton dispersion relation for a homogeneous 3D electron gas. Expressing k_z in terms of ω instead, we get the typical penetration depth $\lambda \propto (\mathrm{Im}\, k_z)^{-1}$ in the homogeneous bulk electron gas as

$$\lambda = \frac{c}{\sqrt{(\omega_p^2 - \omega^2 + c^2 q^2)}}. \tag{12}$$

Notice furthermore from equation (6) that if ϵ_t has a small imaginary part and the real part of ϵ_t is negative, q's inverse gives the decay of the field. The frequency distinguishing propagating from evanescent waves thus corresponds to the solution of $\epsilon_t(q(\omega, k_z)) = 0$.

Equation (6) with the use of equations (7) and (8) does not have a solution for real q and ω for any values of γ/ω. However, it can be solved for a complex q in the form $q = q_1 + \mathrm{i}q_2$. q_2^{-1} then describes the length over which attenuation of the TEM field *amplitude* takes place. To find the penetration depth in the LEG (in the limit $\omega < qc$) we can write equation (6) in the following form for this situation:

$$x^2 + \frac{y}{2(1 + \mathrm{i}z)}\left[\frac{x \sinh x}{\cosh x - \cos(k_z d)}\right] = 0 \tag{13}$$

where we have introduced $x \equiv pd$ and two other important dimensionless ratios:

$$y = \left(\frac{\omega_p d}{c}\right)^2 \tag{14}$$

and

$$z = \gamma/\omega. \tag{15}$$

y and z control the physics of the problem in an obvious way. y compares the geometrical length scale of the system with the typical length scale of the electromagnetic field in an electron gas (c/ω_p). z compares the time-scale (ω^{-1}) of the external electromagnetic field with the relaxation time-scale (γ^{-1}). In all of our applications we will be in a situation where y is very small. For instance for a modulation-doped GaAs/Al$_x$Ga$_{1-x}$As superlattice with $\epsilon_b = 13.1$, $m^* = 0.068 m_e$, $d = 700$ Å, $n_s = 5.54 \times 10^{16}$ m^{-2} and $c = 3 \times 10^8$ m s^{-1} we have $\omega_p = 5.32 \times 10^{13}$ s^{-1} (35 meV) and hence c/ω_p is 5.64×10^{-6} m and therefore y is $1.5 \times 10^{-4} \ll 1$.

We are mainly interested in two limiting cases: (i) $\cos(k_z d) \simeq 1$; all layers of the LEG in phase or the so-called strong-coupling limit [6]; and (ii) $\cos(k_z d) \simeq -1$; neighbouring layers out of phase or the so-called weak-coupling limit [6]. We will mainly focus on the first case since we have primarily long-wavelength external electromagnetic fields in mind. We do however comment on the other limit where appropriate. In the first case where both y and $k_z d$ are very small, x is obviously small (cf. equation (13)) and we can therefore expand the structure factor to arrive at

$$x^2 + \frac{y}{1+\mathrm{i}z}\left[\frac{x^2}{x^2+(k_z d)^2}\right] = 0. \tag{16}$$

For the sake of setting the length scale of the penetration, we consider two extreme values of z, i.e. $z \ll 1$ and $z \gg 1$. For $k_z d = 0$ and $z \ll 1$ we then get a penetration depth ($q = q_1 + \mathrm{i}q_2$):

$$(\mathrm{Im}\, q)^{-1} \equiv \lambda_1 = \frac{c}{\sqrt{(\omega_p^2 - \omega^2)}} \tag{17}$$

which we recognize as being similar to the penetration depth in a bulk solid (cf. equation (12)). Notice furthermore that this result (for $z \ll 1$) is restricted to $\gamma \ll \omega < \omega_p$, where the upper limit is related to the fact that we need to consider an evanescent wave.

In the case where $z \gg 1$ and $k_z d = 0$,

$$\lambda_1 = \frac{c}{\omega_p}\sqrt{(2z)}. \tag{18}$$

This λ_1 is simply the classical skin depth in the case of a bulk metal. This suggests that a LEG responds to a TEM field like a 3D free-electron gas for $k_z d \to 0$. When $k_z d \neq 0$, but small, equation (16) has the solutions $x = 0$ ($\omega = qc$) and $x^2 = -(k_z d)^2 - y/(1+\mathrm{i}z)$. The first of these does not correspond to an evanescent wave (ϵ_t is not negative). Since y is so small, $x \approx \mathrm{i}k_z d$, especially for large z. This means that $\lambda_1 \simeq 1/k_z$ and therefore λ_1 is practically independent of z.

In the case where $\cos(k_z d) \simeq -1$ we notice from equation (13) that, since y is so small, x has to be very close to $\mathrm{i}k_z d$ to increase the structure function accordingly. In other words, also in this limit, $\lambda_1 \approx 1/k_z$ for all values of z, suggesting that λ_1 is actually almost independent of $z(\omega)$ for all k_z except when $k_z d$ is extremely small. k_z in turn is determined by the external probe and how it fits in with the geometry since in general $k_z d$ is not given as simply as an odd multiple of π. Finally it is to be noted that in this out-of-phase case equation (6) describes the long-wavelength limit of the dielectric function of a conducting thin film of thickness d. It therefore can be concluded that the penetration depth in a thin metallic film is almost independent of frequency for a TEM field in the plane of the film and it is related to the perpendicular momentum component of the external probe provided that this is related to the geometrical set-up. Notice furthermore that an isolated 2D sheet does not sustain transverse EM modes [3]. For a finite-thickness sheet, see reference [29].

Equation (6) for the case of two layers per unit cell can be written ($\omega \ll qc$) as

$$x^2 + y_1\left[\frac{(1+\alpha)xS}{2(1+\mathrm{i}z)} - \frac{\alpha x^2 Sf}{4\Omega^2(1+\mathrm{i}z)^2}\right] = 0 \tag{19}$$

where as before $x = qd$ and we have introduced $\Omega = \omega/\omega_{p1}$ and $\alpha = \omega_{p2}^2/\omega_{p1}^2$. z is given above in equation (15) and y_1 is obtained from equation (14) on replacing ω_p by ω_{p1}. In the limit of small $k_z d$, equation (19) takes the form

$$x^2 = \Omega^2(1+\mathrm{i}z)\left[\frac{(k_z d)^2)(1+\mathrm{i}z) + y_1(1+\alpha)}{\Omega_c^2 - \Omega^2(1+\mathrm{i}z)^2}\right] \tag{20}$$

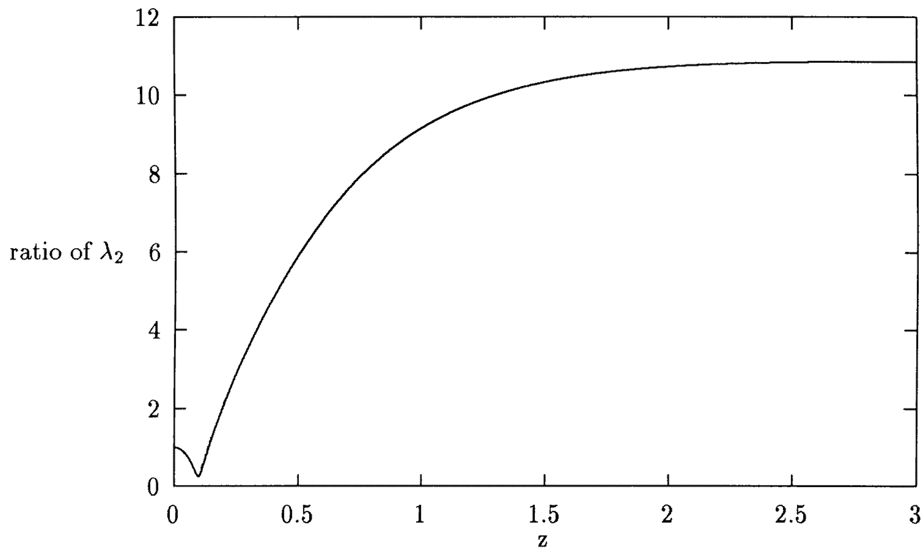

Figure 1. The figure shows the ratio of the penetration depth λ_2, in a system with two layers per unit cell exposed to a transverse electromagnetic field propagating in the plane of the layers, for a small damping $z_c \equiv \gamma/\omega_c = 0.1$ and for a large damping $z_c = 10$. γ is the inverse scattering time and we give the results as a function of $z \equiv \gamma/\omega$. For computing our results shown in this figure we have used parameters corresponding to a modulation-doped GaAs/Al$_x$Ga$_{1-x}$As superlattice. The asymptotic result for this ratio for large z is proportional to $1/z_c$, with the smallest z_c of the two. Notice the structure for very small z, which is the region where we have the possibility of having transverse plasmons propagating in the structure and whose dispersion relation is shown in figure 2.

where we have introduced the characteristic frequency $\omega_c \equiv \Omega_c \omega_{p1}$ given by

$$\omega_c^2 = \frac{\omega_{p1}^2 \omega_{p2}^2}{c^2} d_1(d - d_1) \tag{21}$$

which is very important for the penetration depth as we will find below.

For $z \ll 1$ and $k_z d = 0$ or rather $k_z \ll \omega_p/c$, equation (20) gives the following result for the penetration depth:

$$\lambda_2 \simeq \frac{c}{\surd(\omega_{p1}^2 + \omega_{p2}^2)} \left[1 - \frac{z^2}{z_c^2}\right]^{1/2} \tag{22}$$

where $z_c \equiv \gamma/\omega_c$. On the other hand if $z \gg 1$ and $\cos(k_z d) \simeq 1$ we get

$$\lambda_2 \simeq \frac{c}{\surd(\omega_{p1}^2 + \omega_{p2}^2)} \left[2z\left(1 + \frac{1}{z_c^2}\right)\right]^{1/2}. \tag{23}$$

It can be noted from equation (23) that for $z_c \gg 1$, λ_2 is of the same form as λ_1 in equation (18). For z_c small, λ_2 is much larger than λ_2(large z_c) by a factor $1/z_c$. This clearly shows up in figure 1 where we show a plot of the ratio of λ_2 for a small value of z_c (=0.1) to that of λ_2 for a large value of z_c (=10) for a range of z-values. For the computation of λ_2 we have taken $n_{s1} = n_{s2} = 5.54 \times 10^{16}$ m^{-2}, $d = 1500$ Å and $d_1 = 700$ Å. The rest of the parameters are taken as being the same as those cited after equation (15). These values of the parameters correspond to a GaAs/Al_xGa_{1-x}As superlattice with two successive layers of unequal widths.

Equation (22) suggests that λ_2 can be very small and TEM waves are totally screened at the surface of a medium at $\omega \simeq \omega_c$, for small values of γ. Thus with respect to screening the LEG responds to TEM fields as it responds to a LEM field for $\gamma \leqslant \omega \leqslant \omega_c$. For large z we notice from equation (23) that λ_2 can be very large when there is a small damping (z_c). Furthermore λ_2 is much larger than λ_1 for $\omega \ll \gamma < \omega_c$ suggesting that the LEG is almost transparent for TEM fields in this frequency region.

For $0 < k_z d < 1$, $\lambda_2 \simeq 1/k_z$ while the situation for $\cos k k_z d = -1$ is more complicated and no analytical results can be given except when $z \gg 1$ where $\lambda_2 \simeq d/\pi$.

4. Transverse plasmons

Equation (22) indicates that q_2 becomes imaginary for $\gamma \ll \omega < \omega_c$ ($z_c < z$), suggesting that for this range of ω, equation (6) with the use of equation (9) can have a solution for real q and ω. This implies that there could exist transverse plasmons in the frequency range $\gamma < \omega < \omega_c \ll qc$, which is not found in the situation of one layer per unit cell [3]. However, equation (6) makes use of equation (4) which is valid for $\omega \gg qv_F$. Therefore in order to take into consideration the correct limit of $\epsilon_t(q, k_z, \omega)$ as $\omega \to 0$, we modify the form of $\epsilon_t(q, k_z, \omega)$ in equation (9) by letting $\omega_{pi}^2/\omega(\omega + \mathrm{i}\gamma)$ ($i = 1, 2$) change to $\omega_{pi}^2/[\omega(\omega + \mathrm{i}\gamma) + q^2 v_{Fi}^2/2]$ ($i = 1, 2$) letting γ be the same for simplicity. v_{Fi} ($i = 1, 2$) are the Fermi velocities of charge carriers in layer one and layer two, respectively, of a unit cell. We then solve equation (6) for ω using $\omega_{p1} = \omega_{p2}$ in equation (9) and we obtain

$$\omega_{\pm}^2(q, k_z) = \left(\frac{qv_F}{2}\right)^2 [b \pm \sqrt{(b^2 - 4A(1 + A))}]/(1 + A) \tag{24}$$

where

$$b(q, k_z) \equiv \frac{f(q)}{qa_0^*} - 1 - 2A(q, k_z) \tag{25}$$

and

$$A(q, k_z) \equiv \frac{c^2 qa_0^*}{2v_F^2 S(q, k_z)}. \tag{26}$$

a_0^* is the effective Bohr radius ($\epsilon_b \hbar^2/m^* e^2$) which for our choice of parameters is 102 Å. In the limit of small qd, A is $\mathrm{O}((qd)^2)$ and the first term in b (equation (25)) goes to a constant value G, defined below in equation (28), depending only on the geometry of the problem.

The $\omega_{\pm}$ will be well-behaved plasma modes if $b^2 \geqslant 4A(1 + A)$ and if b is positive. This implies restrictions on qd and $k_z d$ which can only be fulfilled for small values of qd and $k_z d$. We obtain

$$k \equiv \sqrt{(q^2 + k_z^2)} \leqslant \frac{v_F(G - 1)}{c\sqrt{(da_0^* G)}} \equiv k_c \tag{27}$$

where we have introduced

$$G = \frac{2d_1(d - d_1)}{da_0^*}. \tag{28}$$

$G \geqslant 1 + 2A$ in order that b and k_c are positive. This puts restrictions on d_1/d for the superlattices which can show this type of mode, in terms of the value of a_0^*. It should be a

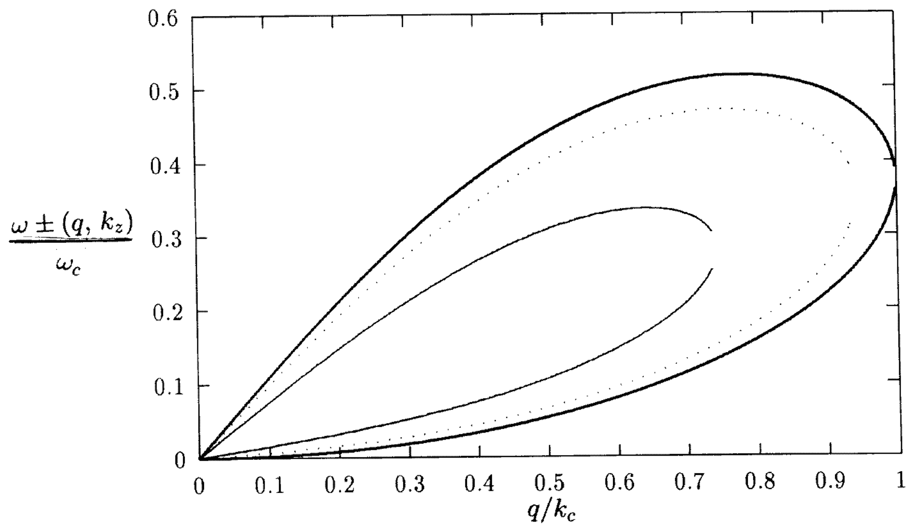

Figure 2. The frequency of the transverse plasmon $\omega_\pm(q, k_z)$ (in units of ω_c, equation (21)) in a layered electron gas with two layers per unit cell plotted as a function of q/k_c with $k_z d$ as a parameter. For the upper three modes (ω_+) $k_z d$ is 0, 1/3 and 2/3 downwards while for the lower three modes (ω_-) it is the other way around. k_c and ω_c define the upper wavenumber and frequency respectively for which these modes can exist. Notice that the ω_+ (upper modes) have a linear dispersion for small q while the ω_- (lower modes) are quadratic. Again the calculation is made with parameters corresponding to a modulation-doped GaAs/$Al_x Ga_{1-x}$As superlattice. Similar modes should also show up in other layered structures, like cuprate superconductors.

feasible experimental test to compare two superlattices, with two layers per unit cell, where one fulfils the condition for having transverse plasmons and the other does not, and to see whether this can be distinguished experimentally. We find that $\omega_\pm$ can be well-behaved transverse plasmons modes in (q, ω)-space when $k \leqslant k_c$ and $\gamma < \omega_\pm < \omega_c < qc$. The ω_c and k_c are calculated using the values of parameters used to compute λ_2 above, and it is found that $\omega_c/\omega_p = 0.01$ where ω_p is now smaller than before, since d is larger; furthermore, $k_c d = 0.03$ and $G = 7.32$. Our computed values of $\omega_\pm/\omega_c$ as a function of q/k_c for three different values of $k_z d$ (0, 1/3 and 2/3), using the values of parameters used to compute λ_2, are shown in figure 2. In terms of our cut-off parameters ω_c and k_c we can write (for $k_z d = 0$)

$$\frac{\omega_\pm^2}{\omega_c^2} = \frac{(G-1)^3}{8G^2}\left(\frac{q}{k_c}\right)^2 \frac{[1-(G-1)q^2/2Gk_c^2 \pm \sqrt{(1-q^2/k_c^2)}]}{[1+(G-1)^2 q^2/4Gk_c^2]}. \tag{29}$$

Notice that the maximum frequency $\omega_\pm$ depends on G: $\omega_\pm(G) = f(G)\omega_c$. However, $f(G)$ is of order unity which is why we use ω_c to normalize $\omega_\pm$.

Figure 2 shows that ω_+ is much larger than ω_- for all values of q and k_z. Also ω_+ is almost linearly dependent on q for $0 \leqslant q \leqslant 0.3k_c$, while ω_- increases almost quadratically for all values of q. Both ω_+ and ω_- go to zero as $q/k_c \to 0$. The slope, or phase velocity, of the ω_+-mode is related to v_F ($[\sqrt{((G-1)/2)}]v_F$). The condition $\gamma \ll \omega_\pm$ can be satisfied in a modulation-doped GaAs/$Al_x Ga_{1-x}$As superlattice which has two unequal successive $Al_x Ga_{1-x}$As layers separated by a GaAs layer. In a modulation-doped GaAs/$Al_x Ga_{1-x}$As superlattice electron–impurity scattering is negligibly small. Therefore the main contribution to γ comes from electron–lattice scattering which could be reduced to a required level by cooling the sample. Though we carried out computations of λ_1 and λ_2 and $\omega_\pm$ for the modulation-doped GaAs/Al_xGa_{1-x}As superlattice, our equations (13), (19) and (24) can also be computed for a normal slab of cuprate superconductors which have a highly anisotropic layered structure. Several spectroscopic studies on the normal state of cuprate superconductors show that $\gamma \propto \omega$ over a wide frequency region of interest and ω_p is much higher than what could be observed in modulation-doped semiconductor superlattices. This suggests that transverse plasmons in the microwave region might also be found in cuprate superconductors like $YBa_2Cu_3O_{7-\delta}$ and $Bi_2Sr_2CaCu_2O_8$ which each have two conducting layers of charge carriers in a unit cell. For a further discussion of the relevant length scales and the influence of tunnelling between layers see reference [30] and a forthcoming publication where we will present a more detailed theoretical analysis of the existence of transverse plasmons in high-T_c materials. Finally we should add the observation that (surface) modes limited to certain frequency and wave-vector intervals, so-called stop modes [31], have been shown to exist in ordinary metallic/insulator superlattices [32] exhibiting a relationship between k_c and G rather similar to that found here.

5. Summary

Summing up, we have found new modes of transverse plasmons in a layered electron gas which are well-behaved plasma modes for $k < k_c$ and $\gamma < \omega_\pm < \omega_c < qc$. The value of $\omega_\pm$ is of the order of a few GHz for typical semiconductor superlattices. They can be observed in systems consisting of two layers of charge carriers in a unit cell, at low temperatures. These plasma modes are basically created by the interlayer interactions between layers in a unit cell. These plasmon modes have not to our knowledge been reported before. Our calculation of the electromagnetic penetration depth with one layer per unit cell, λ_1, shows that for in-phase layers ($k_z d \to 0$) it approaches c/ω_p and $(c/\omega_p)\sqrt{2z}$ for $z \ll 1$ and $z \gg 1$ respectively, while for layers not in phase, λ_1 is almost independent of z $(=\gamma/\omega)$. The behaviour of λ_2 is similar to that of λ_1 for large z and z_c $(=\gamma/\omega_c)$. However for small z, λ_2 significantly differs from λ_1, just as it does for large z and small z_c. It approaches zero for very small z (approaching z_c) and it becomes imaginary for still lower values of z. It is this behaviour which makes possible the existence of transverse plasmons at very low frequencies and small wave vectors.

Acknowledgments

AC Sharma acknowledges the generous and stimulating atmosphere at the Applied Physics Department of the Chalmers University of Technology and Göteborg University. This project was supported by the Swedish Institute, the Swedish Natural Science Research Council and the Department of Science and Technology, New Delhi (India).

References

[1] Quinn J J, Eliasson G and Hawrylak P 1992 *Spatial Dispersion in Solids and Plasmas* ed P Halevi (Amsterdam: Elsevier Science) p 243
[2] Fertig H and Das Sarma S 1990 *Phys. Rev. Lett.* **65** 1482–5; 1991 *Phys. Rev.* B **44** 4480–94
[3] Shi H and Griffin A 1991 *Phys. Rev.* B **44** 11 977–85
[4] Côté R and Griffin A 1993 *Phys. Rev.* B **48** 10 404–25
[5] Stern F 1967 *Phys. Rev. Lett.* **18** 546–8

[6] Fetter A L 1973 *Ann. Phys., NY* **81** 367–93; 1974 *Ann. Phys., NY* **88** 1–25
[7] Sharma A C and Kulshrestha I 1992 *Phys. Rev.* B **46** 6472–6
[8] Griffin A and Pindor A J 1989 *Phys. Rev.* B **39** 11 503–14
[9] Sharma A C 1995 *Solid State Commun.* **95** 569–72
[10] Bloss W L and Brody E M 1982 *Solid State Commun.* **43** 523–8
[11] Das Sarma S and Quinn J J 1982 *Phys. Rev.* B **25** 7603–18
[12] Jain J K and Allen P B 1985 *Phys. Rev. Lett.* **54** 947–50; 1985 *Phys. Rev.* B **32** 997–1004
[13] Giuliani G F and Quinn J J 1983 *Phys Rev. Lett.* **51** 919–22
[14] Das Sarma S and Madhukar A 1981 *Phys. Rev.* B **23** 805–15
[15] Katayama S and Ando T 1985 *J. Phys. Soc. Japan* **54** 1615–26
[16] Tzoar N and Zhang C 1986 *Phys. Rev.* B **34** 1050–6
[17] Sharma A C 1991 *Mod. Phys. Lett.* B **5** 455–63
[18] Kresin V Z and Morawitz H 1987 *Phys. Rev.* B **37** 7854–7
[19] Chang J-J and Scalapino D J 1989 *Phys. Rev.* B **40** 4299–305
[20] Ivanchenko Y M 1993 *Phys. Rev.* B **48** 15 966–76
[21] Vardanian R A 1995 *Phys. Rev.* B **51** 15 286–90
[22] Sharma A C and Sood A K 1994 *J. Phys.: Condens. Matter* **6** 1553–62
[23] Olego D, Pinczuk A, Gossard A C and Wiegmann W 1982 *Phys Rev.* B **25** 7867–70
[24] Sood A K, Menéndez J, Cardona M and Ploog K 1985 *Phys. Rev. Lett.* **54** 2111–4, 2115–8
[25] Sooryakumar R, Pinczuk A, Gossard A and Wiegmann W 1985 *Phys. Rev.* B **31** 2578–80
[26] Scamarcio G, Haines M, Abstreiter G, Molinari E, Baroni S, Fischer A and Ploog K 1993 *Phys. Rev.* B **47** 1483–8
[27] Božović I 1990 *Phys. Rev.* B **42** 1969–84
Božović I, Kim J H, Harris J S Jr and Lee W Y 1991 *Phys. Rev.* B **43** 1169–72
[28] Pines D 1963 *Elementary Excitations in Solids* (New York: Benjamin)
[29] Dahl D A and Sham L J 1977 *Phys. Rev.* B **16** 651–61
[30] Sharma A C and Kulshrestha I 1994 *Physica* C **228** 254–8
[31] Harstein A, Burstein E, Brion J J and Wallis R F 1973 *Surf. Sci.* **34** 81–9
[32] Apell P and Hunderi O 1993 *Phys. Scr.* **48** 494–6